## Mixed Models for Longitudinal Ordinal and Nominal Outcomes

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## Why analyze as ordinal?

- Efficiency: Armstrong & Sloan (1989, Amer Jrn of Epid) and Strömberg (1996, Amer Jrn of Epid) report efficiency losses between 49% to 87% when dichotomizing an ordinal outcome with five categories.
- Bias: continuous model can yield correlated residuals and regressors when used for ordinal outcomes; continuous model does not take into account the ceiling and floor effects of the ordinal outcome. Results in biased estimates of regression coefficients and is most critical when the ordinal variables is highly skewed (see Bauer & Sterba, 2011, Psych Methods)
- Logic: continuous model can yield predicted values outside of the range of the ordinal variable.

Ordinal Logistic Regression Model (aka Proportional Odds or Cumulative Logit Model) - McCullagh (1980)

$$\log\left[\frac{P(Y \le c)}{1 - P(Y \le c)}\right] = \gamma_c - \boldsymbol{x}'\boldsymbol{\beta}$$

 $c = 1, \ldots, C - 1$  for the C categories of the ordinal outcome

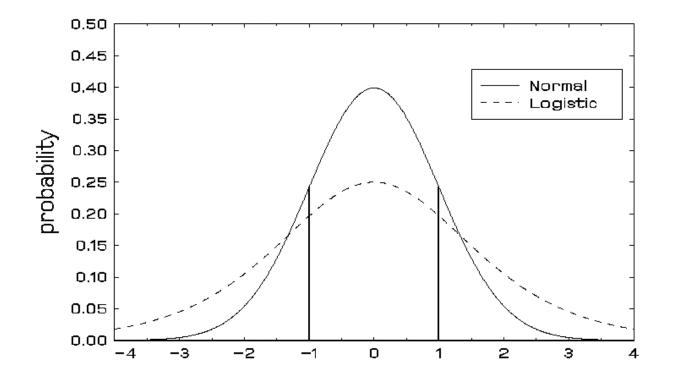
- $\boldsymbol{x}$  = vector of explanatory variables (plus the intercept)
- $\gamma_c$  = threshold parameters; reflect cumulative logits when  $\boldsymbol{x} = 0$ (for identification:  $\gamma_1 = 0$  or  $\beta_0 = 0$ )
- positive association between explanatory variable x and ordinal outcome variable Y is reflected by  $\beta$
- x is assumed to have the same effect on each cumulative logit (proportional odds assumption)

### **Ordinal Response and Threshold Concept**

Continuous  $y_i$  - unobservable latent variable - related to ordinal response  $Y_i$  via "threshold concept"

- threshold values  $\gamma_1, \gamma_2, \ldots, \gamma_{C-1}$  ( $\gamma_0 = -\infty$  and  $\gamma_C = \infty$ )
- C = number of ordered categories

Response occurs in category c,  $Y_i = c$  if  $\gamma_{c-1} < y_i < \gamma_c$ 



## The Threshold Concept in Practice

"How was your day?" (what is your level of satisfaction today?)

• Satisfaction may be continuous, but we sometimes emit an ordinal response:



## Model for Latent Continuous Responses

Consider the model with p covariates for the latent response strength  $y_i$  (i = 1, 2, ..., N):

$$y_i = \boldsymbol{x}_i' \boldsymbol{\beta} + \varepsilon_i$$

- probit:  $\varepsilon_i \sim \text{standard normal (mean=0, variance=1)}$
- logistic:  $\varepsilon_i \sim \text{standard logistic (mean=0, variance} = \pi^2/3)$

 $\Rightarrow \beta$  estimates from logistic regression are larger (in abs. value) than from probit regression by approximately  $\sqrt{\pi^2/3} = 1.8$ 

Underlying latent variable

- useful way of thinking of the problem
- not an essential assumption of the model

## **Mixed-effects ordinal logistic regression model** (Hedeker & Gibbons, 1994, 1996)

- i = 1, ... N level-2 units (clusters or subjects)
- $j = 1, ..., n_i$  level-1 units (subjects or repeated observations)
- $c = 1, 2, \ldots, C$  response categories
- $Y_{ij}$  = ordinal response of level-2 unit *i* and level-1 unit *j*

How was your day? (asked repeatedly each day for a week)



Mixed-effects ordinal logistic regression model

$$\lambda_{ijc} = \log \left[ \frac{P_{ijc}}{(1 - P_{ijc})} \right] = \gamma_c - (\boldsymbol{x}'_{ij}\boldsymbol{\beta} + \boldsymbol{z}'_{ij}\boldsymbol{v}_i)$$

• 
$$P_{ijc} = \Pr\left(Y_{ij} \leq c \mid \boldsymbol{v}; \gamma_c, \boldsymbol{\beta}, \boldsymbol{\Sigma}_{\upsilon}\right) = \frac{1}{1 + \exp(-\lambda_{ijc})}$$

• 
$$p_{ijc} = \Pr(Y_{ij} = c \mid \boldsymbol{v}; \gamma_c, \boldsymbol{\beta}, \boldsymbol{\Sigma}_v) = P_{ijc} - P_{ijc-1}$$

- C-1 strictly increasing model thresholds  $\gamma_c$
- $\boldsymbol{x}_{ij} = p \times 1$  covariate vector
- $\boldsymbol{z}_{ij} = r \times 1$  design vector for random effects
- $\beta = p \times 1$  fixed regression parameters
- $\boldsymbol{v}_i = r \times 1$  random effects for level-2 unit  $i \sim N(\mathbf{0}, \boldsymbol{\Sigma}_v)$

## Model for Latent Continuous Responses

Model with p covariates for the latent response strength  $y_{ij}$ :

$$y_{ij} = \boldsymbol{x}'_{ij}\boldsymbol{\beta} + v_{0i} + \varepsilon_{ij}$$

where  $v_{0i} \sim N(0, \sigma_v^2)$ , and assuming

- $\varepsilon_{ij} \sim$  standard normal (mean 0 and  $\sigma^2 = 1$ ) leads to mixed-effects ordinal probit regression
- $\varepsilon_{ij} \sim$  standard logistic (mean 0 and  $\sigma^2 = \pi^2/3$ ) leads to mixed-effects ordinal logistic regression

Underlying latent variable

- not an essential assumption of the model
- useful for obtaining intra-class correlation (r)

$$r = \frac{\sigma_v^2}{\sigma_v^2 + \sigma^2}$$

and for design effect (d)

$$d = \frac{\sigma_v^2 + \sigma^2}{\sigma^2} = 1/(1-r)$$

ratio of actual variance to the variance that would be obtained by simple random sampling (holding sample size constant)

## Scaling of regression coefficients

Fixed-effects model

 $\pmb{\beta}$  estimates from logistic regression are larger (in abs. value) than from probit regression by approximately

$$\sqrt{\frac{\pi^2/3}{1}} = 1.8$$

because

- $V(y) = \sigma^2 = \pi^2/3$  for logistic
- $V(y) = \sigma^2 = 1$  for probit

## Mixed-effects model

 $\beta$  estimates from mixed-effects (random intercepts) model are larger (in abs. value) than from fixed-effects model by approximately

$$\sqrt{d} = \sqrt{\frac{\sigma_v^2 + \sigma^2}{\sigma^2}}$$

because

- $V(y) = \sigma_v^2 + \sigma^2$  in mixed-effects (random intercepts) model
- $V(y) = \sigma^2$  in fixed-effects model
- difference depends on size of random-effects variance  $\sigma_v^2$
- more complex for models with multiple random effects

## **Treatment-Related Change Across Time**

Data from the NIMH Schizophrenia collaborative study on treatment related changes in overall severity. IMPS item 79, *Severity of Illness*, was scored as:

- 1 = normal or borderline mentally ill
- 2 = mildly or moderately ill
- 3 = markedly ill
- 4 = severely or among the most extremely ill

The experimental design and corresponding sample sizes:

	S	ampl	e s	ize at	t V	Vee	ek	
Group	0	1	2	3	4	5	6	completers
PLC (n=108)	107	105	5	87	2	2	70	65%
DRUG $(n=329)$	327	321	9	287	9	7	265	81%

Drug = Chlorpromazine, Fluphenazine, or Thioridazine

Main question of interest:

• Was there differential improvement for the drug groups relative to the control group?

• Under SSI, Inc > "SuperMix (English)" or "SuperMix (English) Student"



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New Spreadsheet	Ctrl+N
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Import Data File	Ctrl+I

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03								Apply
	(A)_Patient	(B)_Imps79	(C)_Imps79D	(D)_Imps790	(E)_TxDrug	(F)_Week	(G)_SqrtWee (H	_Tx*SWe
1	1103	5.50	1	4	1	0	0.00	0.00 -
2	1103	3.00	0	2	1	1	1.00	1.00
3	1103	-9.00	-9	-9	1	2	1.41	1.41
4	1103	2.50	0	2	1	3	1.73	1.73
5	1103	-9.00	-9	-9	1	4	2.00	2.00
6	1103	-9.00	-9	-9	1	5	2.24	2.24
7	1103	4.00	1	2	1	6	2.45	2.45
8	1104	6.00	1	4	1	0	0.00	0.00
9	1104	3.00	0	2	1	1	1.00	1.00
10	1104	-9.00	-9	-9	1	2	1.41	1.41
11	1104	1.50	0	1	1	3	1.73	1.73
12	1104	-9.00	-9	-9	1	4	2.00	2.00
13	1104	-9.00	-9	-9	1	5	2.24	2.24
14	1104	2.50	0	2	1	6	2.45	2.45
15	1105	4.00	1	2	1	0	0.00	0.00
16	1105	3.00	0	2	1	1	1.00	1.00
17	1105	-9.00	-9	-9	1	2	1.41	1.41
18	1105	1.00	0	1	1	3	1.73	1.73
19	1105	-9.00	-9	-9	1	4	2.00	2.00
20	1105	-9.00	-9	-9	1	5	2.24	2.24
21	1105	-9.00	-9	-9	1	6	2.45	2.45
22	1106	3.00	0	2	1	0	0.00	0.00
23	1106	1.00	0	1	1	1	1.00	1.00
24	1106	-9.00	-9	-9	1	2	1.41	1.41
25	1106	1.50	0	1	1	3	1.73	1.73
26	1106	-9.00	-9	-9	1	4	2.00	2.00
27	1106	-9.00	-9	-9	1	5	2.24	2.24
28	1106	1.00	0	1	1	6	2.45	2.45

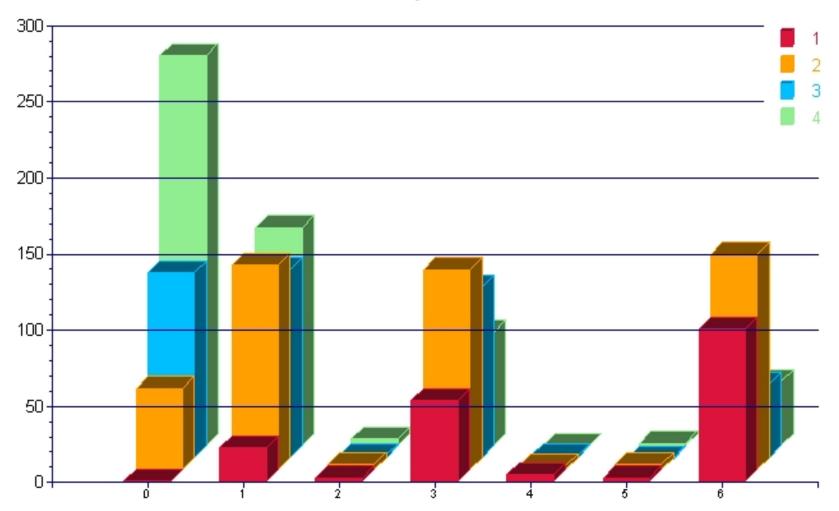
## Select Imps790 column, then "Edit" > "Set Missing Value"

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	1								
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2	1103	3.00		2	1	1	1.00		Missing Value Code: -9
3	1103	-9.00		-9	1	2			· · · · · · · · · · · · · · · · · · ·
4	1103	2.50		2	1	3			OK Cancel
5	1103	-9.00		-9	1	4	2.00		
6	1103	-9.00		-9	1	5			
7	1103	4.00		2	1	6			
8	1104	6.00		4	1	0	0.00		
9	1104	3.00		2		1	1.00		
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17	1105 1105	-9.00		-9		3			
18	1105	-9.00		-9					
19	1105	-9.00		-9		4			

## Select "File" > "Data-based Graphs" > "Bivariate"

List of Variables			
Name	Y	X	
Patient			
mps79			
mps79D			
mps790			
ExDrug			
Week			
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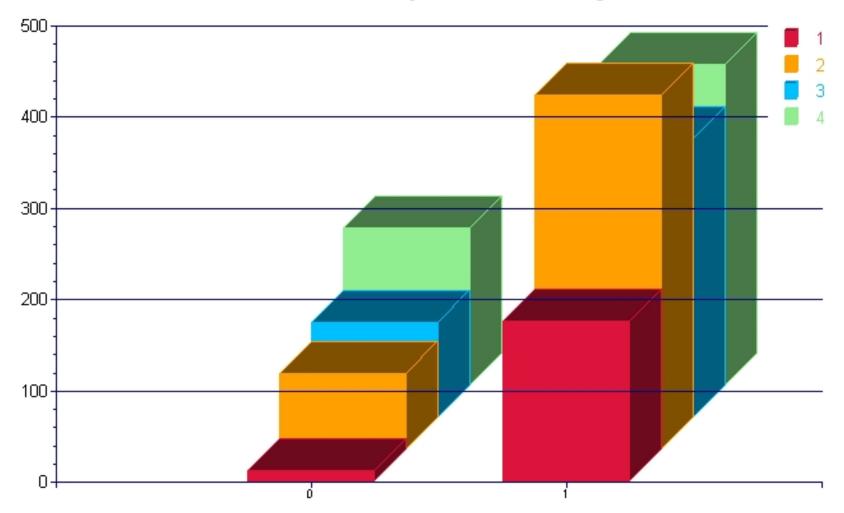
Imps790 vs. Week



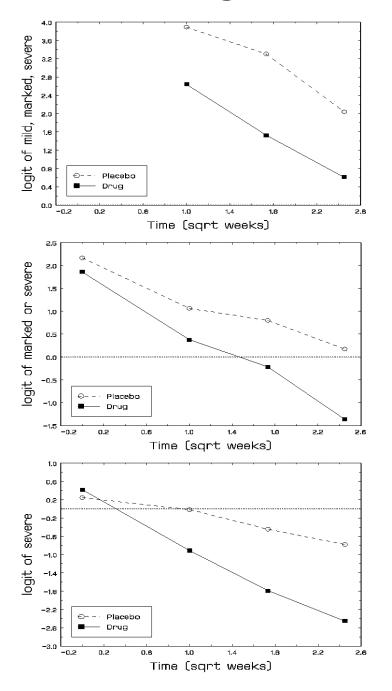
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List of Variables			_
Name	Y	X	
Patient			
Imps79			
Imps79D			
Imps790			
TxDrug		V	
Week			
SqrtWeek			
Tx*SWeek			
C Scatter Plot Line Only Plot Scatter and Line Plu Box and Whisker Bivariate Bar Chart	ot		-

#### Imps790 vs. TxDrug



## **Observed Logits across Time by Condition**



#### Within-Subjects / Between-Subjects components

Within-subjects model - level 1  $(j = 1, ..., n_i \text{ obs})$  $\lambda_{ijc} = \gamma_c - [b_{0i} + b_{1i}\sqrt{Week_j}]$ 

<u>Between-subjects model</u> - level 2 (i = 1, ..., N subjects)

$$b_{0i} = \beta_0 + \beta_2 Grp_i + v_{0i}$$

$$b_{1i} = \beta_1 + \beta_3 Grp_i + v_{1i}$$

 $oldsymbol{v}_i \sim \mathcal{NID}(oldsymbol{0}, oldsymbol{\Sigma}_{arepsilon})$ 

## Under "File" click on "Open Existing Model Setup"

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	Close	
	New Model Setup	Ctrl+W
	Open Existing Model Setup	Ctrl+E
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	Open Text File	
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## Note that "Dependent Variable Type" is "ordered"

Model Setup: schizo2.m	num					
Configuration	<u>i</u> tartin <u>o</u>	g Values   <u>P</u> atterr	ns Adv	anced Linear Transforms		
Title 1: Schiz data - ORDI	NAL	15				
Title 2: Random Intercept	& Trer	nd Model				
Dependent Variable Type:	order	red	•	Level-2 IDs:	Patient	•
Dependent Variable:	Imps	790	-	Level-3 IDs:		-
Categories:	1	Value		Write Bayes Estimates:	no	•
	2	2		Convergence Criterion:	0.0001	
	3	3		Number of Iterations:	100	
Missing Values Present:	true		•	Perform Crosstab	oulation: no	•
Missing Value for the Dep	ender	nt Var: 🕞				
Global Mi	ssing \	/alue: 9		Output Type:	standard	•
Use the arrow k	eys or	click on the desi	red tab to	o select the category of inte	rest for the model.	

## Note the lack of **TxDrug** as an explanatory variable

Model Setup: schizo2 Configuration ⊻ariables		s <u>P</u> atterns <u>A</u> dvanced <u>L</u> inear Tra	ansforms
Available Patient Imps79 Imps79D Imps79D TxDrug Week SqrtWeek Tx*SWeek		Explanatory Variables SqrtWeek Tx*SWeek	L-2 Random Effects SqrtWeek
Select the col	umns of the spre	adsheet to be used as explanatory va	ariables and random effects.

# Make sure "Optimization Method" is set to "adaptive quadrature"

Model Setup: schizo2.mum		
<u>Configuration</u> <u>Variables</u> <u>Starting</u> Values <u>Patterns</u>	vanced	
General Settings Unit Weighting: equal	Explanatory Variable Interacti	
Optimization Method: adaptive quadrature  Number of Quadrature Points: 10		
Ordered Dependent Variable Settings Function Model: logistic Level-2 Random Thresholds: no	Right-Censoring: no	ne 💌
	Model Terms: sul	otract 💌
Use the arrow keys or click on the desired tab	to select the category of interest	for the model.

## Note: Cumulative Logit link function

Seniz data - ORDINAL                 Random Intercept & Trend Model           Model and Data Descriptions         Sampling Distribution         = Multinomial         Link Function       = Cumulative Logit         Number of Level-1 Units       1603         Number of Level-1 Units per Level-2 Unit =       4         4       4       3       4       4         4       2       3       4       4       4       4         4       2       3       4       4       4       4       4         4       2       3       4       4       4       4       4       4         4       4       2       4       4       4       4       4       4         4       4       2       4       4       4       4       4         4       4       2       4       4       4       4       4         4       4       4       4       4       4       4       4       4         4       4       4       4       4       4       4       4       4       4       4       4	schizo2.out												E	
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Link Function       = Cumulative Logit         Number of Level-2 Units       437         Number of Level-1 Units       1603         Number of Level-1 Units       pr Level-2 Unit =         4       4       3       4       4       4       3       4         4       2       3       4       3       4       4       4       3       4         4       2       3       4       3       4       4       4       3       3         2       4       4       4       3       4       4       4       3       3         4       4       2       3       4       4       4       4       3       3         4       4       2       2       4       4       4       3       4         4       4       2       4       4       4       4       3       3         4       4       2       3       4       4       4       4       4       4       4         4       4       4       4       4       4       4       4       4       4       4       4       4       4 <td>Samplin</td> <td>ng Di</td> <td>stribut</td> <td>ion</td> <td></td> <td></td> <td></td> <td>= Multi</td> <td>inomia</td> <td>1</td> <td></td> <td></td> <td></td> <td></td>	Samplin	ng Di	stribut	ion				= Multi	inomia	1				
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Number of Level-1 Units per Level-2 Unit =         4       4       3       4       4       4       4       3       4       4         4       2       3       4       3       4       4       4       4       3       3         2       4       4       4       4       3       4       4       4       4       4         4       4       4       4       3       4       4       4       3       3         2       4       4       4       3       4       4       4       3       3         4       4       2       2       4       5       4       2       4       4       3         4       4       5       4       4       4       4       3       3       4         4       4       5       4       4       4       4       3       3       4         2       3       4       4       4       4       4       4       4       4         4       4       4       4       4       4       4       4       4       4       4 <t< td=""><td></td><td></td><td></td><td>Units</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>				Units										
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		1	. I	3										

Category response indicators (IMPS79O1-IMPS79O4); results of fixed-effects model (to be ignored, or for comparison purposes)

Imps79010.00001.00000.11850.3233Imps79020.00001.00000.29570.4565
Imps7902 0.0000 1.0000 0.2957 0.4565
Imps7903 0.0000 1.0000 0.2570 0.4371
Imps7904 0.0000 1.0000 0.3288 0.4699
SqrtWeek 0.0000 2.4495 1.2204 0.8965
Tx*SWeek 0.0000 2.4495 0.9442 0.9454
o=====================================
o=====================================

#### schizo2.out

Optimization Method: Adaptive	Quadrature
0	0
Number of quadrature points =	10
Number of free parameters =	8
Number of iterations used =	5
-21nL (deviance statistic) =	3325.51347
Akaike Information Criterion	3341.51347
Schwarz Criterion	3384.55053

#### Estimated regression weights

		Standard		
Parameter	Estimate	Error	z Value	P Value
Threshold1	-7.3662	0.3711	-19.8494	0.0000
Threshold2	-3.4647	0.2438	-14.2121	0.0000
Threshold3	-0.8577	0.1850	-4.6353	0.0000
SqrtWeek	-0.8996	0.1897	-4.7430	0.0000
Tx*SWeek	-1.6740	0.2081	-8.0456	0.0000

.

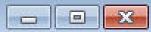
111

#### Odds Ratio and 95% Odds Ratio Confidence Intervals

Parameter	Estimate	Odds Ratio	Lower	Upper	
				and the second second	
Threshold1	-7.3662	0.0006	0.0003	0.0013	
Threshold2	-3.4647	0.0313	0.0194	0.0504	
Threshold3	-0.8577	0.4242	0.2951	0.6096	
SqrtWeek	-0.8996	0.4067	0.2805	0.5899	
Tx*SWeek	-1.6740	0.1875	0.1247	0.2819	
					•

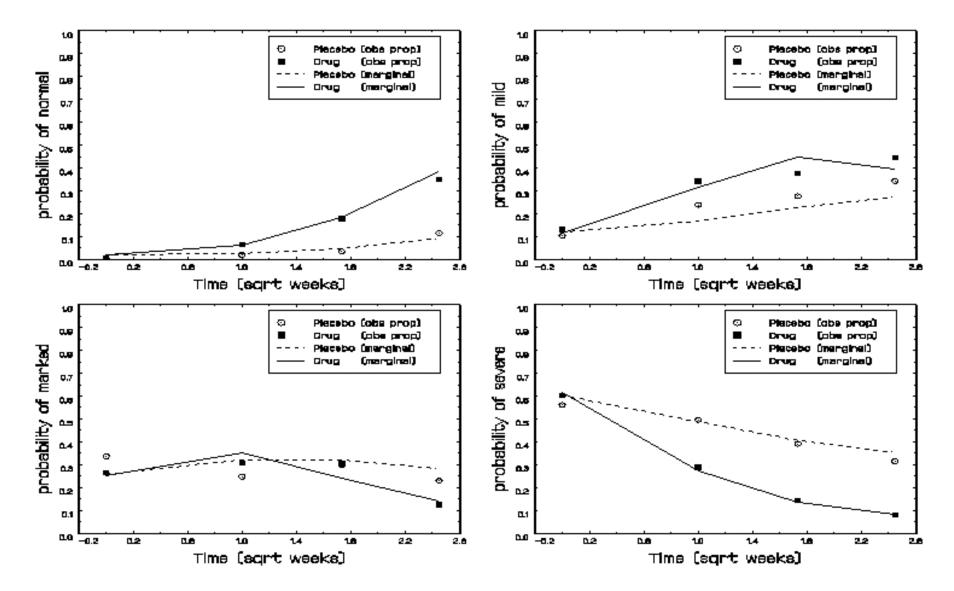
	Estimated leve	el 2 variances a	and covariance	в		
			Standard			
Parameter		Estimate	Error	z Value	P Value	
	ercept					
	ercept :Week					
Sdirmeer/Sdir	week	2.0134	0.41/6	4.0100	0.0000	
Level 2 covari	lance matrix					
	intercept	SqrtWeek				
intercept	7.019260					
SqrtWeek	-1.517353	2.013390				
Level 2 correl	lation matrix					
	intercept	SqrtWeek				
intercept	1.000000					
	-0.403624	1.000000				
	used 1.11 seconds	-				
0========		o				

SCHIZo2.out



Parameter	Estimate	Standard Error	z Value	P Value
Threshold1	-4.5153	0.2582	-17.4851	0.0000
Threshold2	-1.7260	0.1720	-10.0338	0.0000
Threshold3	0.4765	0.1948	2.4459	0.0145
SqrtWeek	-0.8041	0.1469	-5.4753	0.0000
Tx*SWeek	-0.9018	0.1113	-8.1021	0.0000
	nd 95% Odds Ratio Co		Bou	
Odds Ratio am Parameter	nd 95% Odds Ratio Co Estimate	onfidence Interv	Bou	nds Upper
Parameter	Estimate	Odds Ratio	Bour Lower	Upper 
Parameter  Threshold1	Estimate  -4.5153	Odds Ratio 0.0109	Bour Lower  0.0066	Upper  0.0181
Parameter  Threshold1 Threshold2	Estimate -4.5153 -1.7260	Odds Ratio  0.0109 0.1780	Bour Lower  0.0066 0.1270	Upper  0.0181 0.2494
Parameter  Threshold1 Threshold2 Threshold3	Estimate -4.5153 -1.7260 0.4765	Odds Ratio  0.0109 0.1780 1.6104	Bour Lower  0.0066 0.1270 1.0993	Upper  0.0181 0.2494 2.3592
Parameter Threshold1 Threshold2 Threshold3 SqrtWeek	Estimate -4.5153 -1.7260 0.4765 -0.8041	Odds Ratio 0.0109 0.1780 1.6104 0.4475	Bour Lower 0.0066 0.1270 1.0993 0.3356	Upper 0.0181 0.2494 2.3592 0.5967
Parameter  Threshold1 Threshold2 Threshold3	Estimate -4.5153 -1.7260 0.4765	Odds Ratio  0.0109 0.1780 1.6104	Bour Lower  0.0066 0.1270 1.0993	Upper  0.0181 0.2494 2.3592

#### Model Fit of Observed Proportions



**SAS IML code:** SCHZOFIT.SAS - computing marginal probabilities - ordinal model adapted from syntax at http://www.uic.edu/classes/bstt/bstt513/ (Week 12)

TITLE1 'NIMH Schizophrenia Data - Estimated Marginal Probabilities'; PROC IML;

beta = { -.8041, -.9018}; thresh = {-4.5153, -1.726, .4765}; za0 = (thresh[1] - x0\*beta) ; zb0 = (thresh[2] - x0\*beta) ; zc0 = (thresh[3] - x0\*beta) ; za1 = (thresh[1] - x1\*beta) ; zb1 = (thresh[2] - x1\*beta) ; zc1 = (thresh[3] - x1\*beta) ;

```
grp0a = 1 / ( 1 + EXP(- za0));
grp0b = 1 / ( 1 + EXP(- zb0));
grp0c = 1 / ( 1 + EXP(- zc0));
grp1a = 1 / ( 1 + EXP(- za1));
grp1b = 1 / ( 1 + EXP(- zb1));
grp1c = 1 / ( 1 + EXP(- zc1));
```

```
print 'Random intercept and trend model';
print using Population Average Estimates';
print 'marginal prob for group 0 - catg 1' grp0a [FORMAT=8.4];
print 'marginal prob for group 0 - catg 2' (grp0b-grp0a) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 3' (grp0c-grp0b) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 4' (1-grp0c) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 1' grp1a [FORMAT=8.4];
print 'marginal prob for group 1 - catg 2' (grp1b-grp1a) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 3' (grp1c-grp1b) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 4' (1-grp1c) [FORMAT=8.4];
```

## **Proportional and Non-proportional Odds**

Proportional Odds model

$$\log \left[ \frac{P(Y_{ij} \le c)}{1 - P(Y_{ij} \le c)} \right] = \gamma_c - \left[ \boldsymbol{x}'_{ij} \boldsymbol{\beta} + \boldsymbol{z}'_{ij} \boldsymbol{v}_i \right]$$
  
with  $\boldsymbol{v}_i \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_v)$ 

- $\bullet$  relationship between the explanatory variables and the cumulative logits does not depend on c
- effects of  $\boldsymbol{x}$  variables DO NOT vary across the C-1 cumulative logits

Non-Proportional/Partial Proportional Odds model

$$\log\left[\frac{P(Y_{ij} \le c)}{1 - P(Y_{ij} \le c)}\right] = \gamma_{0c} - \left[\boldsymbol{u}_{ij}'\boldsymbol{\gamma}_c + \boldsymbol{x}_{ij}'\boldsymbol{\beta} + \boldsymbol{z}_{ij}'\boldsymbol{\nu}_i\right]$$

 $u_{ij} = h \times 1$  vector for the set of h covariates for which proportional odds is not assumed

- effects of  $\boldsymbol{u}$  variables DO vary across the C-1 cumulative logits
- more flexible model for ordinal response relations
- can be used to empirically test proportional odds assumption

## **Proportional Odds Assumption**: covariate effects are the same across all cumulative logits

		Response		
group	Absent	Mild	Severe	total
control	27	46	27	100
cumulative odds	$\frac{27}{73} = .37$	$\frac{73}{27} = 2.7$		
logit	-1	1		
treatment	38	44	18	100
cumulative odds	$\frac{38}{62} = .61$	$\frac{82}{18} = 4.6$		
logit	5	1.5		

 $\Rightarrow$  group difference = .5 for both cumulative logits

# **Non-Proportional Odds**: covariate effects vary across the cumulative logits

		Response		
group	Absent	Mild	Severe	total
control	27	46	27	100
cumulative odds	$\frac{27}{73} = .37$	$\frac{73}{27} = 2.7$		
logit	-1	1		
treatment	28	60	12	100
cumulative odds	$\frac{28}{72} = .39$	$\frac{88}{12} = 7.3$		
logit	95	2		

 $\Rightarrow$  UNEQUAL group difference across cumulative logits

## Open C:\SuperMixEn Examples\Workshop\Ordinal\schizo2np.mum

(or C:\SuperMixEn Student Examples\Workshop\Ordinal\schizo2np.mum)

No. Col	11.1		0 =
rganize 🔻 New fol	der		
Бежер	Name	Date modified Type	Size
Downloads	📄 schizo1	5/15/2006 2:45 PM MUM File	6 KB
🖳 Recent Places	📄 schizo2	1/30/2013 10:35 AM MUM File	5 KB
💔 Dropbox	📄 schizo2np	3/11/2009 4:02 PM MUM File	3 KB
	SD01	5/21/2007 3:55 PM MUM File	5 KB
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📕 Computer			
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😨 Units (\\SPH-File			
File	name: schizo2np	✓ Mixed Up	Models (*.mum)

#### Note that "Dependent Variable Type" is "ordered"

Model Setup: schizo2np	o.mum		
Configuration	<u>6</u> tarting Values <u>P</u> atterns ,	Advanced Linear Transforms	1
Title 1: Schiz data - ORDI	NAL - NON PROPORTION/	AL ODDS	
Title 2: Random Intercept	& Trend Model		7
Dependent Variable Type:	ordered	Level-2 IDs:	Patient
Dependent Variable:	Imps790	- Level-3 IDs:	<b>_</b>
Categories:	Value	Write Bayes Estimates:	no
	1 1 2 2	Convergence Criterion:	0.0001
	3 3	Number of Iterations:	100
Missing Values Present:	true	<ul> <li>Perform Crosstat</li> </ul>	oulation: no 💌
Missing Value for the Dep	endent Var: 9		
Global Mi	ssing Value: -9	Output Type:	standard 💌
Use the arrow k	eys or click on the desired to	ab to select the category of inte	erest for the model.
1			

## Two explanatory variables: SqrtWeek and Tx\*SWeek

Model Setup: schizo2np.mum		
<u>Configuration</u> <u>Variables</u> <u>Starting</u> Values <u>P</u> al	tterns   <u>A</u> dvanced   <u>L</u> inear Transfo	orms
Available E   Patient   Imps79   Imps790   TxDrug   Week   SqrtWeek   SqrtWeek	Explanatory Variables SqrtWeek Tx*SWeek	L-2 Random Effects SqrtWeek
Select the columns of the spreadshe	et to be used as explanatory variab	les and random effects.

### "Explanatory Variable Interactions" - both are selected

🥂 Model Setup: schizo2np.mum		
Configuration Variables Starting Values Patterns	vanced Linear Transforms	1
General Settings Unit Weighting: equal	Explanatory Variable Inter Include Interactions: Number of Interactions:	yes 💌
Optimization Method: adaptive quadrature  Number of Quadrature Points: 10		
Ordered Dependent Variable Settings Function Model: logistic Level-2 Random Thresholds: no	Right-Censoring:	none
	Model Terms:	subtract
Use the arrow keys or click on the desired tab t	to select the category of inte	rest for the model.

0		==============			
Optimization M	ethod: Adaptive	Quadrature			
0=======					
Number of quadrat	ure points =	10			
Number of free pa	rameters =	12			
Number of iterati	ons used =	6			
-21nL (deviance s	tatistic) =	3324.16153			
Akaike Informatio	n Criterion	3348.16153			
Schwarz Criterion		3412.71711			
	Estimated reg	ression weights			
		Standard			
Parameter	Estimate	Error	z Value	P Value	
Thresholdl	-7.4687	0.4860	-15.3681	0.0000	
Threshold2	-3.5799	0.2779	-12.8808	0.0000	
Threshold3	-0.8100		-4.1848	0.0000	
SqrtWeek	-0.9506	0.3307	-2.8741	0.0041	
Tx*SWeek	-1.6827	0.2879	-5.8456	0.0000	
Interactions of p	redictors with:	Threshold2			
SqrtWeek	-0.0826	0.2991	-0.2761	0.7825	
Tx*SWeek	0.0799	0.2496	0.3199	0.7490	
Interactions of p	redictors with:	Threshold3			
SqrtWeek	0.1171	0.3332	0.3515	0.7252	
Tx*SWeek	-0.0004	0.2870	-0.0013	0.9990	

Proportional Odds Assumption Accepted:  $\chi_4^2 = 3325.51 - 3324.16 = 1.35$ 

#### Linear Transforms

Fixed part of model:

$$\begin{array}{l} \lambda_c \,=\, \hat{\gamma}_{0c} - \left[ \hat{\beta}_1 \texttt{SqrtWeek} + \hat{\beta}_2 \texttt{Tx*SWeek} \right. \\ \left. + \,\, \hat{\gamma}_{1c} \texttt{SqrtWeek} + \hat{\gamma}_{2c} \texttt{Tx*SWeek} \right] \end{array}$$

		cumulative logit	
variable	1 vs 2,3,4	1,2 vs 3,4	1,2,3, vs 4
SqrtWeek	$\hat{eta}_1$	$\hat{eta}_1+\hat{\gamma}_{12}$	$\hat{\beta}_1 + \hat{\gamma}_{13}$
Tx*SWeek	$\hat{eta}_2$	$\hat{eta}_2 + \hat{\gamma}_{22}$	$\hat{\beta}_2 + \hat{\gamma}_{23}$

 $H_0: \beta_1 + \gamma_{12} = 0$ ; SqrtWeek effect is 0 on the 2nd cumulative logit

$$z = \frac{\hat{\beta}_1 + \hat{\gamma}_{12}}{SE(\hat{\beta}_1 + \hat{\gamma}_{12})}$$

Model Setup: schizo2np.mum		- • 💌 🖉	Model Setup: sc	hizo2np.mum				
Configuration   ⊻ariables   Starting \	/alues   <u>Patterns</u>   <u>A</u> dvanced <u>L</u> inear Transforms	1	Configuration   ⊻aria	ables Starting \	Values   <u>P</u> atterns   <u>A</u>	dvanced <u>L</u> inea	ar Transforms	
Linear Transforms SgrtWeek Thresh2	Add Transform		Linear Transf SgrtWeek Thresh2		Add Transform			
Tx*SWeek Thresh2	Copy <u>I</u> ransform		Tx*SWeek Thresh		Copy <u>T</u> ransform	1		
SqrtWeek Thresh3 Tx*SWeek Thresh3	<u>Remove Transform</u>		SqrtWeek Thresh3 Tx*SWeek Thresh		<u>R</u> emove Transform	<u> </u>		
Explanatory Variables: Value SqrtWeek 1 Tx*SWeek	Level-2 Random Effect (Co)variances: Value intercept variance intercept, SqrtWeek SqrtWeek variance		Explanatory Variabl SqrtWeek Tx*SWeek	es: Value 1	Level-2 Random F intercept varian intercept, SqrtW SqrtWeek varian	Value ce eek	Ces:	
Thresholds:	Threshold Interactions:		Thresholds:		Threshold Interac	tions:		
Value	Thresh 2 Thresh 3			Value		Thresh 2	Thresh 3	
1	SqrtWeek 1		1		SqrtWeek			
2	Tx*SWeek		2		Tx*SWeek	1		
3			3					
Select t	he linear transform to review and edit its components. Type to change the transform's name in place.				he linear transform to Type to change the tr			

🛃 Model Setup: schizo2np.mum	📝 Model Setup: schizo2np.mum		
Configuration   Variables   Starting Values   Patterns   Advanced Linear Transforms	Configuration Variables Starting	Values   Patterns   Advanced Linear Transforms	
Linear Transforms       Add Transform         SqrtWeek Thresh2       Copy Iransform         SqrtWeek Thresh3       Bernove Transform         Tx*SWeek Thresh3       Level-2 Random Effect (Co)variances:         Explanatory Variables:       Level-2 Random Effect (Co)variances:         SqrtWeek 1       intercept variance         Tx*SWeek       SqrtWeek	Linear Transforms SqrtWeek Thresh2 Tx*SWeek Thresh2 SqrtWeek Thresh3 Tx*SWeek Thresh3 Explanatory Variables: SqrtWeek Tx*SWeek 1	Add Transform Copy Iransform Bemove Transform Level-2 Random Effect (Co)variances: Value intercept variance intercept, SqrtWeek SqrtWeek variance	
Thresholds: Threshold Interactions:	Thresholds:	Threshold Interactions:	
Value     Thresh 2     Thresh 3       1     SqrtWeek     1       2     Tx"SWeek     1	Value 1 2 3	Thresh 2     Thresh 3       SqrtWeek	
Select the linear transform to review and edit its components. Type to change the transform's name in place.		he linear transform to review and edit its components. Type to change the transform's name in place.	

	     (Gen 	TESTING OF TRANSFORMS eral Linear Hypothesis T				
Coeffi	cients		Estimate	Trans	form No.	
				1	2	3
l Thr	esholdl			0.0000	0.0000	0.0000
5 ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) (	eshold2		-3.57991	0.0000	0.0000	0.0000
	eshold3		-0.80996	0.0000	0.0000	0.0000
4 Sqr			-0.95058	1.0000	0.0000	1.0000
$5 Tx^*$			-1.68268	0.0000	1.0000	0.0000
6 Sqr		*Threshold2	-0.08259	1.0000	0.0000	0.0000
7 Tx*		*Threshold2	0.07985	0.0000	1.0000	0.0000
8 Sqr	tWeek	*Threshold3	0.11712	0.0000	0.0000	1.0000
9 Tx*		*Threshold3	-0.00037	0.0000	0.0000	0.0000
10 Var	(intercep	t)	7.37046	0.0000	0.0000	0.0000
11 Cov	(SqrtWeek	,intercept)	-1.67047	0.0000	0.0000	0.0000
12 Var	(SqrtWeek	)	2.03449	0.0000	0.0000	0.0000
	orm Estim	ate		-1.0332	-1.6028	-0.8335
Standa	rd Error			0.2240	0.2287	0.2063
Z-Stat				-4.6133	-7.0090	-4.0392
Exceed	ence Poba	bility		0.0000	0.0000	0.0001

😤 schizo2np.out

Coefficients	Estimate Transform No. 4	
l Thresholdl	-7.46875 0.0000	
2 Threshold2	-3.57991 0.0000	
3 Threshold3	-0.80996 0.0000	
4 SqrtWeek	-0.95058 0.0000	
5 Tx*SWeek	-1.68268 1.0000	
6 SqrtWeek *Threshold2	-0.08259 0.0000	
7 Tx*SWeek *Threshold2	0.07985 0.0000	
8 SqrtWeek *Threshold3	0.11712 0.0000	
9 Tx*SWeek *Threshold3	-0.00037 1.0000	
10 Var(intercept)	7.37046 0.0000	
11 Cov(SqrtWeek, intercept)	-1.67047 0.0000	
12 Var(SqrtWeek)	2.03449 0.0000	
Transform Estimate	-1.6831	
Standard Error	0.2370	
Z-Statistic	-7.1012	
Exceedence Pobability	0.0000	

NIMH Schiz Study: Severity of Illness $(N = 437)$									
Ordinal LR Estima	Ordinal LR Estimates (se) - random intercept and trend r								
	Proportional	Non-Proportional Odds Model							
	Odds Model								
		1 vs 2,3,4	1,2 vs 3,4	1,2,3 vs 4					
Time (sqrt week)	-0.900	-0.951	-1.033	-0.834					
	(0.190)	(0.331)	(0.224)	(0.206)					
Drug by Time	-1.674	-1.683	-1.603	-1.683					
	(0.208)	(0.288)	(0.229)	(0.237)					
$-2\log L$	3325.51		3324.16						

• Proportional Odds accepted ( $\chi_4^2 = 3325.51 - 3324.16 = 1.35$ )

#### San Diego Homeless Research Project (Hough)

- 361 mentally ill subjects who were homeless or at very high risk of becoming homeless
- 2 conditions: HUD Section 8 rental certificates (yes/no)
- baseline and 6, 12, and 24 month follow-ups
- Categorical outcome: housing status
  - -streets / shelters (Y = 0)
  - community / institutions (Y = 1)
  - independent (Y = 2)

*Question:* Do Section 8 certificates influence housing status across time?

• Under SSI, Inc > "SuperMix (English)" or "SuperMix (English) Student"



• Under "File" click on "Open Spreadsheet"

New Spreadsheet	Ctrl+N
	Cuitti
Open Spreadsheet	Ctrl+O
Import Data File	Ctrl+I

• Open C:\SuperMixEn Examples\Workshop\Nominal\SDHOUSE.ss3 (or C:\SuperMixEn Student Examples\Workshop\Nominal\SDHOUSE.ss3)

Open Spreadsheet	X
Look in: 🌗 Nominal	▼ 🗢 🗈 🖝
Name	Date modified T
SDHOUSE.ss3	5/17/2007 11:24 PM S
<	4
File name: SDHOUSE	Open
	✓ Cancel

#### $C:\SuperMixEn\ Examples\Workshop\Nominal\SDHOUSE.ss3$

												Apply
	(A)_ID	(B) HOUSIN	(C)_SECTIO	(D) TIME1	(E) TIME2	(F) TIME3	(G) SECT8T (	H) SECT8T (	I) SECT8T3	(J) NOSECT	(K)_TIME	(L) SEC8TI
1 [		1 1	1	. ~ 0	0	0	0	0	~0	0	0	0
2		1 2	1	1	0	0	1	0	0	0	1	1
3		1 2	1	0	1	0	0	1	0	0	2	2
4		1 2	1	0	0	1	0	0	1	0	3	3
5		2 1	1	0	0	0	0	0	0	0	0	0
6		2 2	! 1	1	0	0	1	0	0	0	1	1
7	:	2 2	! 1	0	1	0	0	1	0	0	2	2
8		2 1	1	0	0	1	0	0	1	0	3	3
9		3 0	1	0	0	0	0	0	0	0	0	0
10		3 2	! 1	1	0	0	1	0	0	0	1	1
11		3 2	! 1	0	1	0	0	1	0	0	2	2
12		3 2	! 1	0	0	1	0	0	1	0	3	3
13		4 1	1	0	0	0	0	0	0	0	0	0
14		4 1	1	1	0	0	1	0	0	0	1	1
15		4 1	1	0	1	0	0	1	0	0	2	2
16		4 1	1	0	0	1	0	0	1	0	3	3
17		5 0	1	0	0	0	0	0	0	0	0	0
18		5 1	1	1	0	0	1	0	0	0	1	1
19		5 2		0	1	0	0	1	0	0	2	2
20		5 2	! 1	0	0	1	0	0	1	0	3	3
21		6 2	! 1	0	0	0	0	0	0	0	0	0
22		6 2		1	0	0	1	0	0	0	1	1
23		6 2		0	1	0	0	1	0	0	2	2
24		6 2		0	0	1	0	0	1	0	3	3
25		7 2		0	0	0	0	0	0	0	0	0
26		7 2		1	0	0	1	0	0	0	1	1
27		7 2		0	1	0	0	1	0	0	2	2
28		7 2		0	0	1	0	0	1	0	3	3
29		8 2		0	0	0	0	0	0	0	0	0
30		8 C	· · · · · · · · · · · · · · · · · · ·	1	0	0	1	0	0	0	1	1
31		8 C		0	1	0	0	1	0	0	2	2
32		8 2	! 1	0	0	1	0	0	1	0	3	3

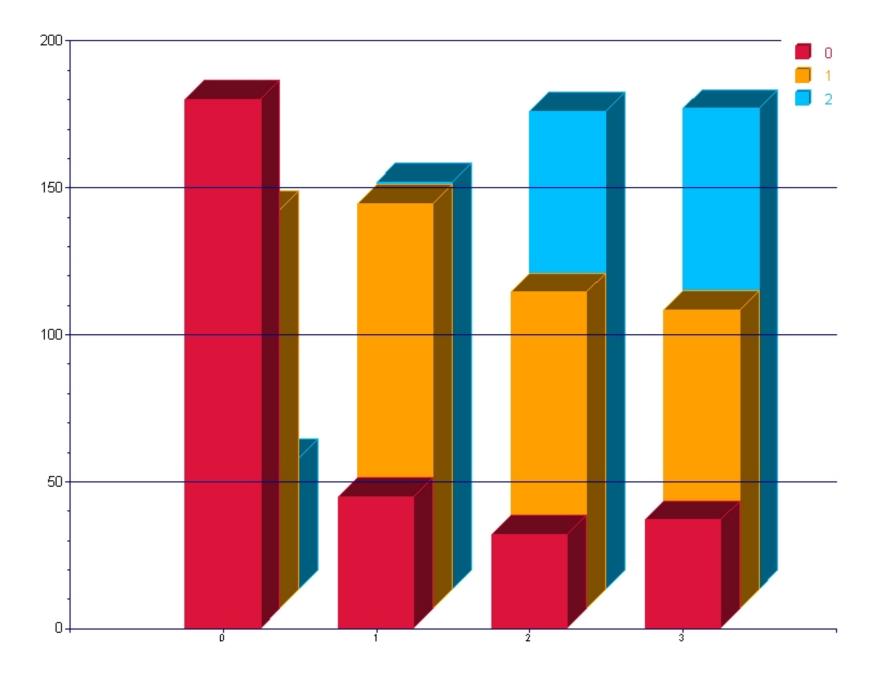
#### Select Housing column, then "Edit" > "Set Missing Value"

	(A)_ID	(B)_HOUSIN (C)	SECTIO	(D) TIME1	(E)_TIME2	(F) TIME3	(G) SECT8T	(H)_SECT8T (	I) SECT8T3	(J) NOSECT	(K)_TIME (L	_]_SEC8TI
121	31	2	- 1	0	0	0	0		~0	0	0	- 0
122	31	2	1	1	0	0	1	0	0	0	1	1
123	31	2				0	0	1	0	0	2	2
124	31	2	M	issing Value Co	-de: 999	1	0	0	1	0	3	3
125	32	0	IVI	issing value ci	uue. Joog	0	0	0	0	0	0	0
126	32	0			-	. 1 0	1	0	0	0	1	1
127	32	999		OK	Cance	0	0	1	0	0	2	2
128	32	999			0	1	0	0	1	0	3	3
129	33	1	1	0	0	0	0	0	0	0	0	0
130	33	999	1	1	0	0	1	0	0	0	1	1
131	33	999	1	0	1	0	0	1	0	0	2	2
132	33	999	1	0	0	1	0	0	1	0	3	3
133	34	2	1	0	0	0	0	0	0	0	0	0
134	34	2	1	1	0	0	1	0	0	0	1	1
135	34	0	1	0	1	0	0	1	0	0	2	2
136	34	0	1	0	0	1	0	0	1	0	3	3
137	35	0	1	0	0	0	0	0	0	0	0	0
138	35	1	1	1	0	0	1	0	0	0	1	1
139	35	0	1	0	1	0	0	1	0	0	2	2
140	35	0	1	0	0	1	0	0	1	0	3	3
141	36	2	1	0	0	0	0	0	0	0	0	0
142	36	2	1	1	0	0	1	0	0	0	1	1
143	36	2	1	0	1	0	0	1	0	0	2	2
144	36	2	1	0	0	1	0	0	1	0	3	3
145	37	0	1	0	0	0	0	0	0	0	0	0
146	37	1	1	1	0	0	1	0	0	0	1	1
147	37	1	1	0	1	0	0	1	0	0	2	2
148	37	999	1	0	0	1	0	0	1	0	3	3
149	38	1	1	0	0	0	0	0	0	0	0	0
150	38	999	1	1	0	0	1	0	0	0	1	1
151	38	2	1	0	1	0	0		0	0	2	2
152	38	1	1	0	0	1	0		1	0	3	3

### Select "File" > "Data-based Graphs" > "Bivariate"

	Y	X	
Name	T		_
HOUSING			
SECTION8			
TIME1			
TIME2			
TIME3			
SECT8T1			
SECT8T2			
SECT8T3			
NOSECT8			
TIME	Г	<b>V</b>	
SEC8TIME			
C Scatter Plot C Line Only Plot C Scatter and Line Plot C Box and Whisker			

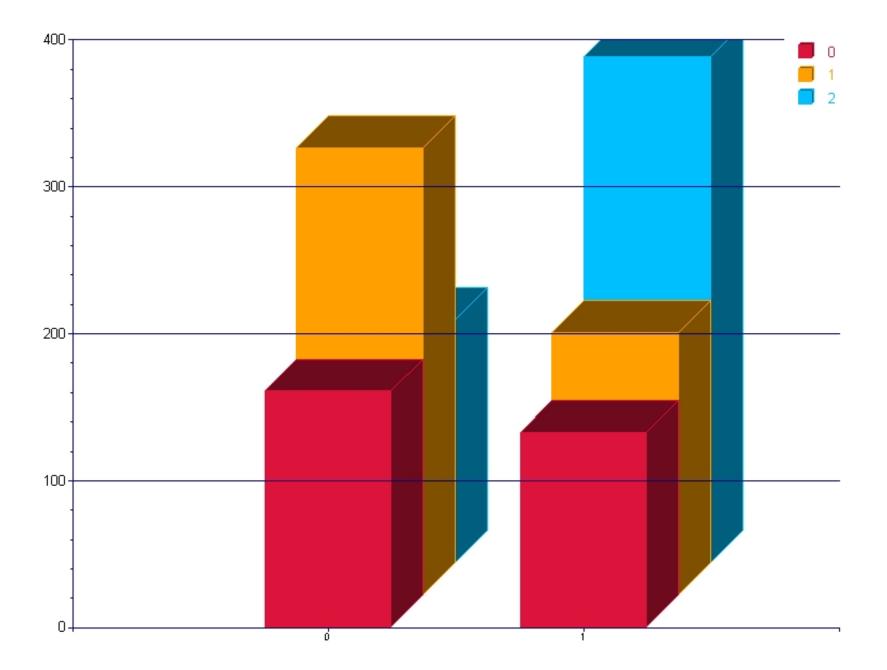
#### HOUSING vs. TIME

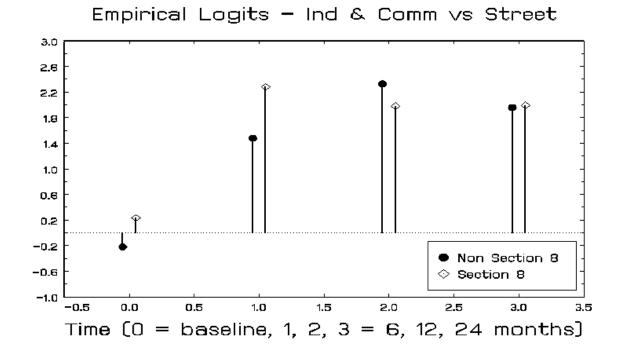


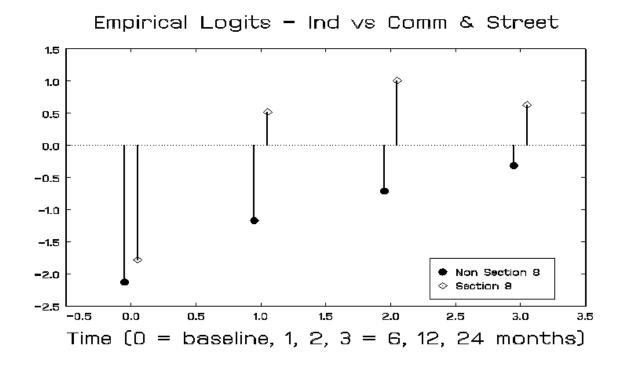
### Select "File" > "Data-based Graphs" > "Bivariate"

Name	Y		. <b>.</b>
ID		X	-
HOUSING	<u>v</u>		
SECTION8			
TIME1	Г	Г	
TIME2			
TIME3		Γ	
SECT8T1			
SECT8T2			
SECT8T3			
NOSECT8			
TIME			
SEC8TIME			-
C Scatter Plot C Line Only Plot C Scatter and Line Plot C Box and Whisker			

#### HOUSING vs. SECTION8







Under "File" click on "Open Existing Model Setup"

File	) Edit Window Help	
	New Project	Ctrl+N
	Import Data File	Ctrl+I
	Close	
	New Model Setup	Ctrl+W
	Open Existing Model Setup	Ctrl+E
	New Syntax File	
	Open Syntax File	
	Open Text File	
	Data-based Graphs	
	Open Graph	Ctrl+G
	Save	Ctrl+S
	Save As	
	Exit	

Open C:\SuperMixEn Examples\Workshop\Ordinal\SDO1.mum (or C:\SuperMixEn Student Examples\Workshop\Ordinal\SDO1.mum)

Name	Date modified	-	
	- are mounted	Туре	Size
SD01	5/21/2007 4:55 PM	MUM File	5 KB
SDO2	5/21/2007 4:57 PM	MUM File	3 KB
TVOC	5/18/2007 12:15 AM	MUM File	5 KB
TVOS	5/18/2007 12:16 AM	MUM File	5 KB
Tvosc	5/18/2007 12:16 AM	MUM File	5 KB
	SDO2 TVOC TVOS	SD02         5/21/2007 4:57 PM           TVOC         5/18/2007 12:15 AM           TVOS         5/18/2007 12:16 AM	SD02         5/21/2007 4:57 PM         MUM File           TVOC         5/18/2007 12:15 AM         MUM File           TVOS         5/18/2007 12:16 AM         MUM File

### Note that "Dependent Variable Type" is "ordered"

Model Setup: SDO1.mu	m					x
Configuration	<u>S</u> tartin <u>c</u>	Values   <u>P</u> atterr	ns   <u>A</u> dv	anced Linear Transforms		
Title 1: San Diego Homele	essnes	s Study - ORDIN.	AL			
Title 2: Logistic link functi	on					
Dependent Variable Type:	order	ed	•	Level-2 IDs:	ID	•
Dependent Variable:	HOU	SING	-	Level-3 IDs:		<b>-</b>
Categories:		Value		Write Bayes Estimates:	no	•
	1	0		Convergence Criterion:	0.0001	
	3	2		Number of Iterations:	100	
Missing Values Present:	true		•	Perform Crosstab	ulation: no	•
Missing Value for the Dep	enden	t Var: 999				
Global Mi	ssing V	alue: 999		Output Type:	standard	•
Select the form o	of the d	lependent variab	le. The c	options on the screens will c	hange as required.	

## All explanatory variables are indicator (dummy) variables

Available     E     2       ID     ID       HOUSING	Model Setup: SDO1.n	<u>P</u> atterns   <u>A</u> dvanced   <u>L</u> inear Transf	orms
HOUSING       I         SECTION8       I         TIME1       I         TIME2       I         TIME2       I         TIME3       SECT8T1         SECT8T1       I         SECT8T2       I         SECT8T3       I         NOSECT8       I         TIME       I         SEC8TIME       I	ID HOUSING SECTION8 TIME1 TIME2 TIME3 SECT8T1 SECT8T1 SECT8T2 SECT8T3 NOSECT8 TIME	SECTION8 TIME1 TIME2 TIME3 SECT8T1 SECT8T2	

Housing status across time: 1289 observations within 361 subjects Ordinal Mixed Regression Model estimates and standard errors (se)

	Proportion	al Odds		Non	-Proportio	onal (	Ddds	
			Non-stre	$eet^1$	Independ	$lent^2$	differen	nce
term	estimate	se	estimate	se	estimate	se	estimate	se
$threshold_1$	.220	.198	.322	.207				
$threshold_2$	2.966	.230			2.700	.298		
t1 (6 month)	1.736	.235	2.298	.303	1.079	.343	-1.219	.408
t2 (12 month)	2.316	.247	3.346	.387	1.645	.340	-1.701	.467
t3 (24  month)	2.500	.253	2.822	.348	2.145	.337	676	.422
section 8 (yes=1)	.497	.277	.592	.294	.323	.394	269	.384
section 8 by $t1$	1.409	.341	.566	.467	2.024	.471	1.457	.581
section 8 by $t2$	1.173	.354	958	.506	2.017	.476	2.975	.600
section 8 by $t3$	.638	.349	366	.480	1.073	.464	1.440	.573
subject var	2.134	.354	2.128	.353	$(ICC \approx$	<b>:</b> .4)		
$-2\log L$	2274.39		2222.25		$\chi_7^2 = 52.14$	1)		

**bold** indicates p < .05 *italic* indicates .05

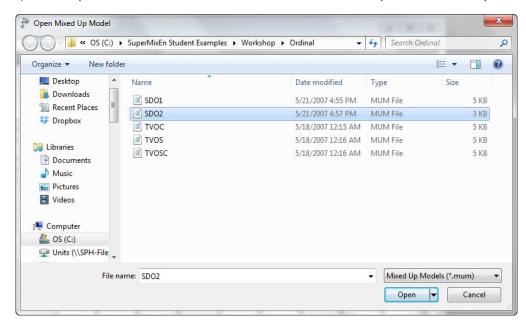
 $^{1} =$ independent + community vs street

 $^{2}$  = independent vs community + street

For Non-Proportional Odds model, under "File" click on "Open Existing Model Setup"

P s	uperMix	
File	Edit Window Help	
	New Project	Ctrl+N
	Import Data File	Ctrl+I
	Close	
	New Model Setup	Ctrl+W
	Open Existing Model Setup	Ctrl+E
	New Syntax File	
	Open Syntax File	
	Open Text File	
	Data-based Graphs	+
	Open Graph	Ctrl+G
	Save	Ctrl+S
	Save As	
	Exit	

Open C:\SuperMixEn Examples\Workshop\Ordinal\SDO2.mum (or C:\SuperMixEn Student Examples\Workshop\Ordinal\SDO2.mum)



### Note that "Dependent Variable Type" is "ordered"

Model Setup: SDO2.mu	m		
Configuration Variables	<u>S</u> tarting Values   <u>P</u> atterr	ns   <u>A</u> dvanced   <u>L</u> inear Transfor	ms
Title 1: San Diego Homele	essness Study - ORDIN.	AL	
Title 2: non-proportional o	dds model		7
Dependent Variable Type:	ordered	Level-2 ID	s: ID 💌
Dependent Variable:	HOUSING	▼ Level-3 ID	Is:
Categories:	Value	Write Bayes Estimate	es: no
	1 0 2 1	Convergence Criterio	on: 0.0001
	3 2	Number of Iteration	ns: 100
Missing Values Present:	true	Perform Cross	atabulation: no
Missing Value for the Dep	endent Var: 999		
Global Mi	ssing Value: 999	Output Typ	e: standard 💌
Use the arrow k	eys or click on the desi	red tab to select the category of	interest for the model.

#### Note "Explanatory Variable Interactions" is set to 7

Model Setup: SDO2.mum	
<u>Configuration</u> <u>Variables</u> <u>Starting</u> Values <u>Patterns</u>	vanced Linear Transforms
General Settings Unit Weighting: equal	Explanatory Variable Interactions Include Interactions: yes Number of Interactions: 7
Optimization Method: adaptive quadrature Number of Quadrature Points: 10	
Ordered Dependent Variable Settings Function Model: logistic Level-2 Random Thresholds: no	Right-Censoring: none
	Model Terms: subtract

Model Setup: SDO2.mum	- • •	Model Setup: SDO2.mum
Configuration Variables Starting Values Patterns Advanced Linear Transforms		<u>C</u> onfiguration <u>V</u> ariables <u>S</u> tarting Values <u>P</u> atterns <u>A</u> dvanced <u>L</u> inear Transforms Linear Transforms Add Transform
Time1 Thresh2     Copy Iransform       [] Eemove Transform ]		Time1 Thresh2     Copy Iransform       Time2 Thresh2     Eemove Transform
Explanatory Variables: Level-2 Random Effect (Co)variances:           Value         Value           TIME1         1           TIME2         intercept variance           TIME3         SECT8T1		Explanatory Variables:     Level-2 Random Effect (Co)variances:       Image: TIME1     Image: TIME2       TIME2     Image: TIME3       SECT8T1     Image: TIME2
Thresholds:         Threshold Interactions:           Value         Value           1         TIME1	<u> </u>	Thresholds: Threshold Interactions:       Value     Value       1     TIME1
2 TIME2 TIME3 SECT8T1		2 TIME2 11 TIME3 SECT8T1
Deletes the currently selected transform: Time1 Thresh2.		Enter Threshold Interactions for the transform Time2 Thresh2.

Model Setup: SDO2.mum		Model Setup: SDO2.mum 📃 📼 💌
Configuration   Variables   Starting Values   Patterns   Advanced Linear Transforms		Configuration   Variables   Starting Values   Patterns   Advanced Linear Transforms
Linear Transforms     Add Transform       Time3 Thresh2     Copy Iransform       Sect8T1		Linear Transforms     Add Transform       Sect8T1     Copy Iransform       Sect8T3     Eemove Transform
Explanatory Variables: Level-2 Random Effect (Co)variances:           Value         Value           SECT8T1         intercept variance           SECT8T2         sectars           SECT8T3         1           SECTION8         Image: Sectars		Explanatory Variables:     Level-2 Random Effect (Co)variances:       Value     Value       SECT8T1     intercept variance       SECT8T2     SECT8T3       SECT10N8     I
Thresholds: Threshold Interactions:		Thresholds: Threshold Interactions:
Value     Value       1     TIME3       2     SECT8T1       SECT8T2     SECT8T3       Enter Threshold Interactions for the transform Sect8T3.	• •	Value     Value       1     SECT8T1       2     SECT8T2       SECTION8     1       Enter Threshold Interactions for the transform Section8.
1		1

#### Mixed Multinomial Logistic Regression Model

 $Y_{ij}$  = nominal response of level-2 unit *i* and level-1 unit *j* 

Which member of The Polkaholics is your favorite? (asked before, during, and after a show)



Mixed-effects Multinomial Logistic Regression Model

$$\log \frac{p_{ijc}}{p_{ij1}} = \boldsymbol{u}'_{ij}\boldsymbol{\gamma}_c + \boldsymbol{z}'_{ij}\boldsymbol{v}_{ic} \qquad c = 2, 3, \dots C$$

• C - 1 contrasts to reference cell (c = 1)

- $\bullet$  regression effects  $\boldsymbol{\gamma}_c$  vary across contrasts
- $\bullet$  random-effects  $oldsymbol{v}_{ic}$  vary across contrasts
  - independent

- correlated

For example, with 
$$C = 3$$

contrast	ordinal	nominal
c1	2 & 3 vs 1	2  vs  1
c2	3 vs 1 & 2	3  vs  1

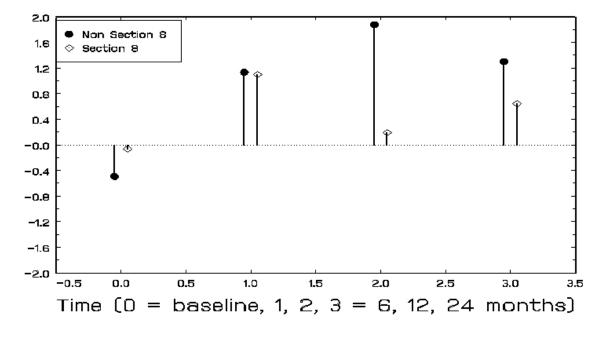
#### Model in terms of the category probabilities

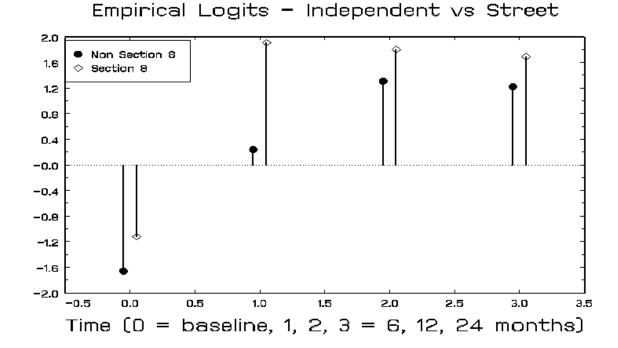
$$p_{ijc} = \Pr(Y_{ij} = c \mid \boldsymbol{v}_{ic}) = \frac{\exp(z_{ijc})}{1 + \boldsymbol{\Sigma}_{h=2}^{C} \exp(z_{ijh})} \quad \text{for } c = 2, 3, \dots, C$$

$$p_{ij1} = \Pr(Y_{ij} = 1 \mid \boldsymbol{v}_{ic}) = \frac{1}{1 + \boldsymbol{\Sigma}_{h=2}^{C} \exp(\boldsymbol{z}_{ijh})}$$

where the multinomial logit  $z_{ijc} = u'_{ij} \gamma_c + z'_{ij} v_{ic}$ 

Empirical Logits - Community vs Street





Under "File" click on "Open Existing Model Setup"

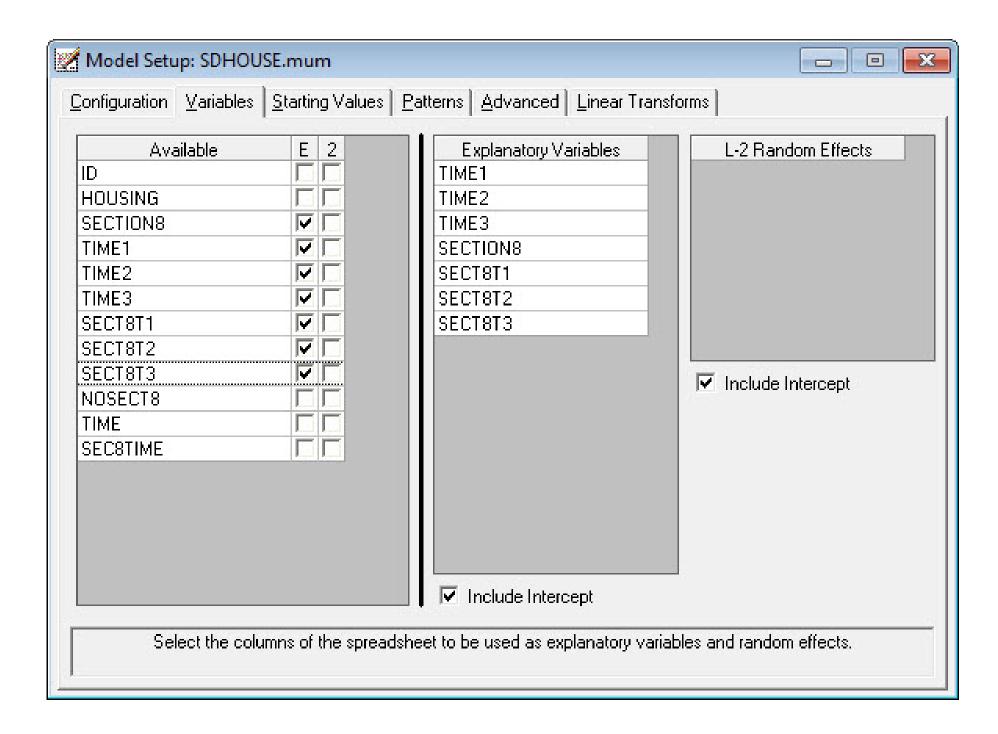
File	) Edit Window Help	
	New Project	Ctrl+N
	Import Data File	Ctrl+I
	Close	
	New Model Setup	Ctrl+W
	Open Existing Model Setup	Ctrl+E
	New Syntax File	
	Open Syntax File	
	Open Text File	
	Data-based Graphs	•
	Open Graph	Ctrl+G
	Save	Ctrl+S
	Save As	
	Exit	

Open C:\SuperMixEn Examples\Workshop\Nominal\sdhouse.mum (or C:\SuperMixEn Student Examples\Workshop\Nominal\sdhouse.mum)

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### Note that "Dependent Variable Type" is "nominal"

Model Setup: SDHOUSE	i.mum		
Configuration	<u>6</u> tarting Values   <u>P</u> atterns   <u>A</u>	dvanced Linear Transforms	1
Title 1: San Diego Homele	essness Study - NOMINAL		
Title 2: random intercept r	nodel - street as reference cel		
Dependent Variable Type:	nominal	Level-2 IDs:	ID 💌
Dependent Variable:	HOUSING	Level-3 IDs:	-
Categories:	Value 1 0 2 1	Write Bayes Estimates: Convergence Criterion:	no 💌
	2 1 3 2	Number of Iterations:	
Missing Values Present:	true	Perform Crosstab	oulation: no
Missing Value for the Dep	endent Var: 999		
Global Mi	ssing Value: 999	Output Type:	standard 💌



#### Can select first or last category as the reference cell

Model Setup: SDHOUSE.mum	
Configuration Variables Starting Values Patterns Advanced Linear Transforms	
General Settings Unit Weighting: equal	
Optimization Method: adaptive quadrature  Number of Quadrature Points: 10	
Nominal Dependent Variable Settings Reference Category: first	
Select whether the first or last category of the outcome should be used as the referenc	e category.

## Try independent random effects first

Model Setup: SDHOUSE.mum	
Configuration Variables Starting Values Patterns Advanced Linear Transforms	
Explanatory Variables: Level-2 (Co)variance Patterns:	
Select the type of pattern for the level-2 random effects.	

#### SDHOUSE.out

Opti	.mizat	ion	Metho	d: N	umerical	Quadrature	-
Trank							~
Number	-			-		1	.0
Number Number	-			-		100	0.8

-21nL (deviance statistic) =	2218.71755
Akaike Information Criterion	2254.71755
Schwarz Criterion	2347.62674

#### Estimated regression weights

- - -

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Parameter	Estimate	Error	z Value	P Value	
Response Code 1	vs Code 0				
intercept	-0.4511	0.1847	-2.4425	0.0146	
TIME1	1.9421	0.3002	6.4685	0.0000	
TIME2	2.8207	0.3838	7.3502	0.0000	
TIMES	2.2604	0.3548	6.3709	0.0000	
SECTIONS	0.5207	0.2618	1.9892	0.0467	
SECT8T1	-0.1350	0.4617	-0.2925	0.7699	
SECT8T2	-1.9180	0.5167	-3.7122	0.0002	
SECT8T3	-0.9525	0.4808	-1.9811	0.0476	
Response Code 2	vs Code 0				
	vs Code 0 	0.3791	-7.0560	0.0000	
Response Code 2  intercept TIME1			-7.0560		
intercept			6.1550	0.0000	
intercept TIME1	 -2.6746 2.6847	0.4362	6.1550 8.2208	0.0000	
intercept TIME1 TIME2	-2.6746 2.6847 4.0907	0.4362 0.4976	6.1550 8.2208 8.7231	0.0000 0.0000 0.0000	
intercept TIME1 TIME2 TIME3	-2.6746 2.6847 4.0907 4.1009	0.4362 0.4976 0.4701	6.1550 8.2208 8.7231 1.6225	0.0000 0.0000 0.0000 0.1047	
intercept TIME1 TIME2 TIME3 SECTION8	-2.6746 2.6847 4.0907 4.1009 0.7816	0.4362 0.4976 0.4701 0.4817	6.1550 8.2208 8.7231 1.6225 3.2311	0.0000 0.0000 0.1047 0.0012	
intercept TIME1 TIME2 TIME3 SECTION8 SECT8T1	-2.6746 2.6847 4.0907 4.1009 0.7816 2.0009	0.4362 0.4976 0.4701 0.4817 0.6193	6.1550 8.2208 8.7231 1.6225 3.2311 0.8389	0.0000 0.0000 0.0000 0.1047 0.0012 0.4015	
intercept TIME1 TIME2 TIME3 SECTION8 SECT8T1 SECT8T2	-2.6746 2.6847 4.0907 4.1009 0.7816 2.0009 0.5460 0.3035	0.4362 0.4976 0.4701 0.4817 0.6193 0.6509	6.1550 8.2208 8.7231 1.6225 3.2311 0.8389	0.0000 0.0000 0.0000 0.1047 0.0012 0.4015	
intercept TIME1 TIME2 TIME3 SECTION8 SECT8T1 SECT8T2	-2.6746 2.6847 4.0907 4.1009 0.7816 2.0009 0.5460	0.4362 0.4976 0.4701 0.4817 0.6193 0.6509	6.1550 8.2208 8.7231 1.6225 3.2311 0.8389	0.0000 0.0000 0.0000 0.1047 0.0012 0.4015	4

SDHOUSE.out Estimated level 2 variances and covariances . Standard Error z Value P Value Parameter Estimate \_\_\_\_\_ \_\_\_\_\_ 0.7617 0.3280 2.3219 0.0202 intercept1/intercept1 intercept2/intercept2 5.4514 1.0237 5.3255 0.0000 Level 2 covariance matrix intercept1 intercept2 intercept1 0.761654 intercept2 0.000000 5.451441 Level 2 correlation matrix intercept1 intercept2 intercept1 1.000000 intercept2 0.000000 1.000000 Calculation of intracluster correlation (01) \_\_\_\_\_ residualvariance = pi\*pi / 3 (assumed) cluster variance = 0.7617 intracluster correlation = 0.7617 / ( 0.7617 + (pi\*pi/3)) = 0.188 Calculation of intracluster correlation (02) \_\_\_\_\_ residualvariance = pi\*pi / 3 (assumed) cluster variance = 5.4514 intracluster correlation = 5.4514 / ( 5.4514 + (pi\*pi/3)) = 0.624 -111 • b. Save As... Close

### Now allow the random effects to be correlated

Model Setup: SDHOUSE.mum	
Configuration Variables Starting Values Patterns Advanced Linear Transforms	
Explanatory Variables: Level-2 (Co)variance Patterns:	
Select the type of pattern for the level-2 random effects.	

SDHOUSE.out					
0=========					
Optimization Me		Quadrature			
0=======		o			
Number of guadratu	re points =	10			
Number of free par	-	19			
-	Number of iterations used =				
-21nL (deviance st	atistic) =	2180.93648			
	Akaike Information Criterion				
Schwarz Criterion		2218.93648 2317.00729			
	Estimated reg	ression weights			
		Standard			
Parameter	Estimate	Error	z Value	P Value	
Response Code 1 v	rs Code 0				
intercept	-0.6222	0.2330		0.0076	
TIME1	2.3744	0.3477	6.8292	0.0000	
TIME2	3.3452	0.4416	7.5759	0.0000	
TIME3	2.5894	0.4022	6.4378	0.0000	
SECTIONS	0.6516	0.3281		0.0471	
SECT8T1	-0.3311	0.5228	-0.6334	0.5265	=
SECT8T2	-2.4746	0.5897		0.0000	
SECT8T3	-1.1596	0.5438	-2.1322	0.0330	
Response Code 2 v	rs Code 0				
intercept	-2.5987	0.3837	-6.7729	0.0000	
TIME1	2.8878	0.4541	6.3591	0.0000	
TIME2	4.3939	0.5301	8.2887	0.0000	
TIME3	4.3104	0.4958	8.6944	0.0000	
SECTIONS	0.8314	0.4938	1.6836	0.0923	
SECT8T1	1.9688	0.6436	3.0592	0.0022	
SECT8T2	0.2872	0.6769	0.4243	0.6713	
SECT8T3	0.2017	0.6442	0.3130	0.7543	-
•	Ш				•
Save As Close					

LR test comparing models:  $\chi_1^2 = 2218.72 - 2180.94 = 37.78$ 

SDHOUSE.out - • × Estimated level 2 variances and covariances . Standard Error z Value P Value Estimate Parameter -----\_\_\_\_\_ \_\_\_\_\_ intercept1/intercept1 2.7025 0.6968 3.8786 0.0001 intercept2/intercept1 2.8846 0.7649 3.7710 0.0002 intercept2/intercept2 5.8229 1.1358 5.1269 0.0000 Level 2 covariance matrix intercept1 intercept2 intercept1 2.702462 intercept2 2.884597 5.822889 Level 2 correlation matrix intercept1 intercept2 intercept1 1.000000 intercept2 0.727170 1.000000 Calculation of intracluster correlation (01) \_\_\_\_\_ residualvariance = pi\*pi / 3 (assumed) cluster variance = 2.7025 intracluster correlation = 2.7025 / ( 2.7025 + (pi\*pi/3)) = 0.451 Calculation of intracluster correlation (02) \_\_\_\_\_ Ξ residualvariance = pi\*pi / 3 (assumed) cluster variance = 5.8229 intracluster correlation = 5.8229 / ( 5.8229 + (pi\*pi/3)) = 0.639 Ŧ. Þ. 1 III Save As... Close 79

#### Summary

Models for longitudinal ordinal and nominal data as developed as models for continuous and dichotomous data

- Proportional odds models
- Non and partial proportional odds models
- Nominal models (with reference-cell contrasts)
- Grouped-time survival analysis models
- $\Rightarrow$  SuperMix can do it all, including 3-level models, also for counts
  - full likelihood solution using adaptive quadrature